The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1876

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ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

[1]Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

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1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [6,7].

$$\frac{\partial}{\partial t}\left[\rho\left(e+|\vec{u}|^2/2\right)\right]+\nabla\cdot\left[\rho\left(h+|\vec{u}|^2/2\right)\vec{u}\right]=-\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\left(d_{ji}u_i\right) \quad (1.1)$$

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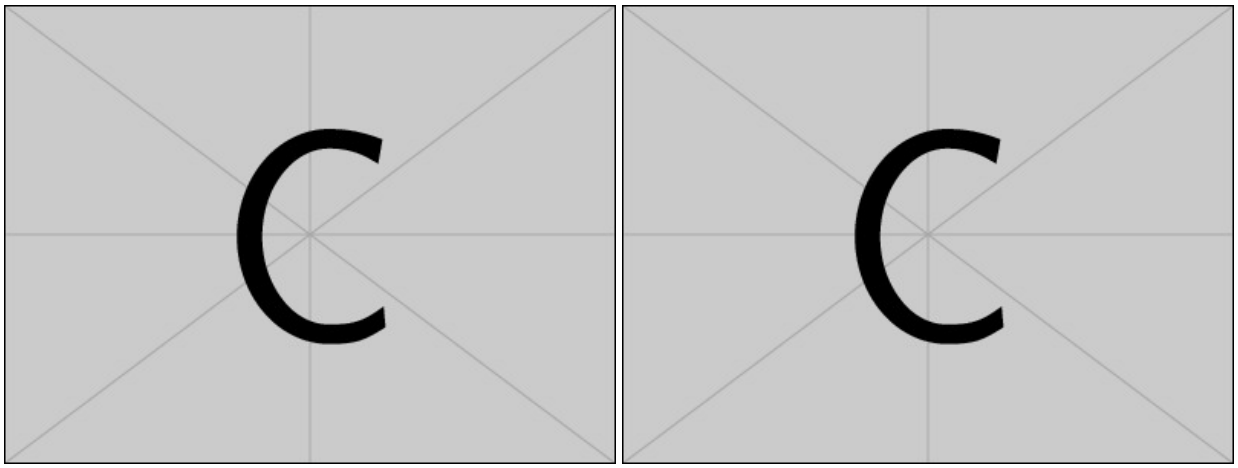

(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Second subfigure

Figure 1.1: A figure with two subfigures

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1.1.1 Subsection eqn. (1.2)

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A subsubsection

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$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1,\varepsilon_1)} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2,\varepsilon_1)} & \frac{\varphi}{(\varphi_2,\varepsilon_2)} & 0 & \cdots & \cdots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3,\varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3,\varepsilon_2)} & \frac{\varphi}{(\varphi_3,\varepsilon_3)} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1},\varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1},\varepsilon_2)} & \cdots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1},\varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1},\varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n,\varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n,\varepsilon_2)} & \cdots & \cdots & \frac{\varphi k_{n,n-1}}{(\varphi_n,\varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n,\varepsilon_n)} \end{pmatrix} \tag{1.2}$$

1.2 Description of our paradigm

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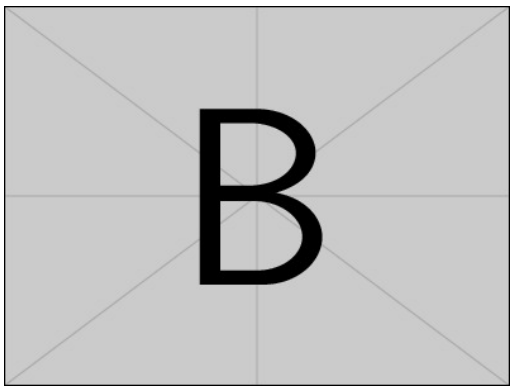

Figure 1.2: Caption text [8].

eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. No dissertation is complete without footnotes.[1,2,3]

1.2.1 Conversion to a metaheuristic

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Subsection

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[1] First footnote: $a_h = F_m$. See section 1.4.
[2] Another interesting detail.
[3] And another really important idea to have in mind [12–17].

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1.3 Other generalizations

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1.3.1 The most general case

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$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \cdots}}}}} \tag{1.3}$$

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application of Stokes theorem leads to the *Kelvin–Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt} \oint_C \mathbf{u} \cdot d\mathbf{r} \tag{1.4}$$

$$= \oint_C \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{r} + \underbrace{\oint_C \mathbf{u} \cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \tag{1.5}$$

$$= \iint_S \nabla \times \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{A} \tag{1.6}$$

$$= \iint_S \nabla p \times \nabla \left(\frac{1}{\rho}\right) \cdot d\mathbf{A} \tag{1.7}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom currents going oppositely.

Weak shock wave A weak shock wave travelling in a perfect gas is described by the following Burgers equation derived from approximations due to G. I. Taylor (1910)

$$u_t + \left(a_0 + \tfrac{1}{2}(\gamma + 1)u\right) u_x = \frac{\delta}{2} u_{xx} \tag{1.8}$$

Table 1.1: The error function and complementary error function

x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$	x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11979
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Here, γ is the specific heat capacity ratio, δ is the diffusivity of sound, u_1 is the downstream speed, $u_2 < u_1$ is the upstream speed, and a_0 is the undisturbed sound speed. The travelling-wave, or Taylor shock, solution of this equation is

$$\frac{u-u_2}{u_1-u_2} = \frac{1}{2}\left[1-\tanh\left(\frac{\gamma+1}{4\delta}(u_1-u_2)(x-ct)\right)\right] \tag{1.9}$$

where $c = a_0 + (\gamma+1)(u_1+u_2)/4$. This solution exists for a compressive wave, $u_1 > u_2$.

Boundary layer model K. O. Friedrichs introduced the following model equation to illustrate a boundary layer (1941):

$$\varepsilon\frac{d^2y}{dx^2} + \frac{dy}{dx} = 1+2x, \quad \text{with } y(0)=0 \text{ and } y(1)=1 \tag{1.10}$$

As $\varepsilon \to 0$, the highest derivative is lost: the equation becomes first order and cannot meet two boundary conditions. This fact results in a boundary layer at one side of the domain, with a thickness $\mathcal{O}(\varepsilon)$. Matching the outer and inner expansions gives the uniform approximation

$$y_{\text{unif}} = e^{-x/\varepsilon} - 1 + x + x^2 + 2\varepsilon\Big[(1-x) - e^{-x/\varepsilon}\Big], \tag{1.11}$$

with $y - y_{\text{unif}} = \mathcal{O}(\varepsilon^3)$ uniformly on $0 \le x \le 1$ as $\varepsilon \to 0$. As it happens, all higher-order terms of both expansions vanish, so y_{unif} is correct to all orders in ε. A comparison to the exact solution shows the error to be subdominant, $\mathcal{O}(e^{-1/\varepsilon})$.

Table 1.2: Table with more complicated columns: $\Delta T_m = \delta\theta/\Theta_0$

Experiment	u [m/s]	T [°C]
The first test we ran this morning	124.3	68.3
The second test we ran this morning	82.50	103.46
Our competitor's test	72.321	141.384

Table 1.3: Brinkman number at which liquid dissipation makes $< 5\%$ or $< 1\%$ contribution to θ_b or q_w from the Graetz solution at the points where θ_b equals 0.05, 0.02, or 0.01

			|Br|			
θ_b	x^+	$q_w D_h/k\Delta T$	5% of θ_b	1% of θ_b	5% of q_w	1% of q_w
0.05	0.09621	0.3770	3.646×10^{-3}	7.292×10^{-4}	1.571×10^{-3}	3.142×10^{-4}
0.02	0.1266	0.1508	1.458×10^{-3}	2.917×10^{-4}	6.284×10^{-4}	1.257×10^{-4}
0.01	0.1496	0.07541	7.292×10^{-4}	1.458×10^{-4}	3.142×10^{-4}	6.284×10^{-5}

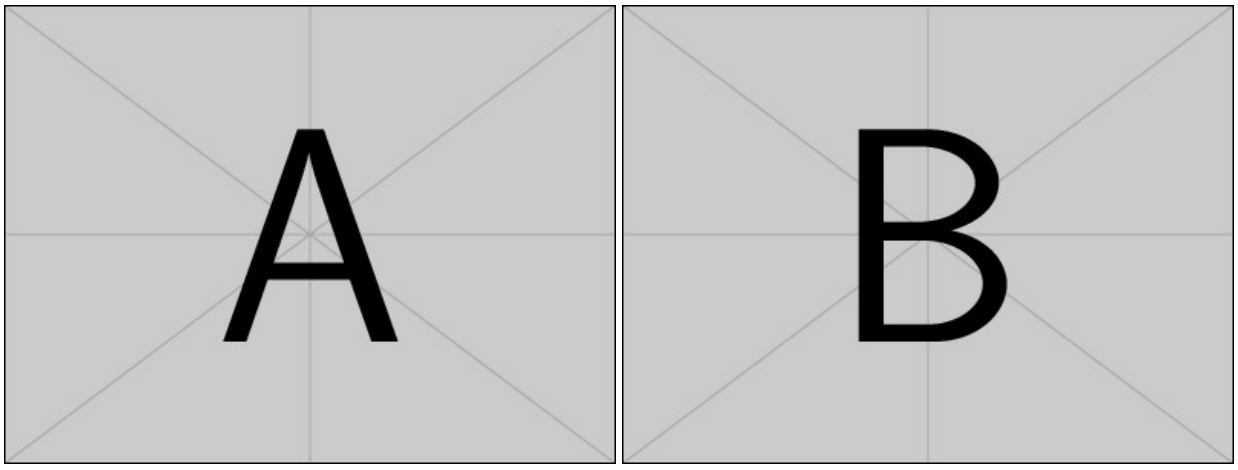
1.5 Further examples of multipanel figures

Examples are given in Figs. 1.3, 1.4, and 1.5. Or consider Figs. 1.4(a), 1.4(b), 1.4(c), and 1.4(d).

1.6 Summary

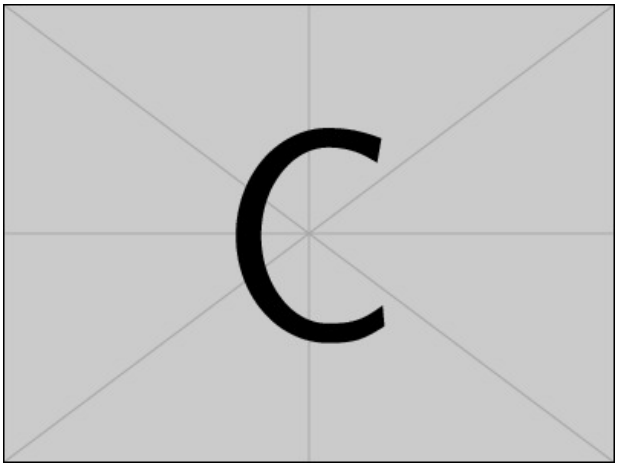
Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra



(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Short caption

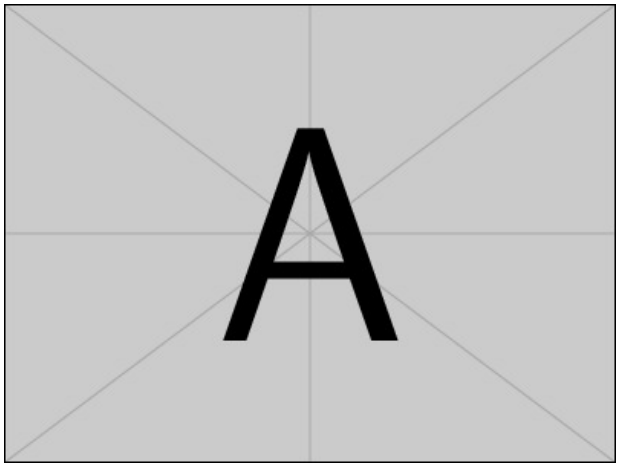


(c) Caption with math, $\mathrm{Re}_d = 100$, and a reference to Fig. 1.3(a).

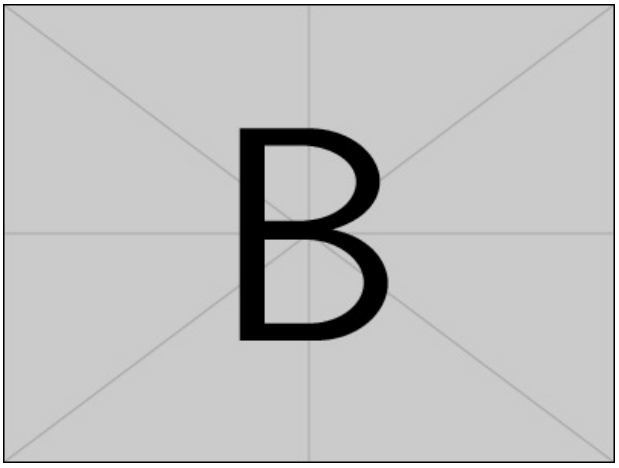
Figure 1.3: A figure with three subfigures: two above one.

gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

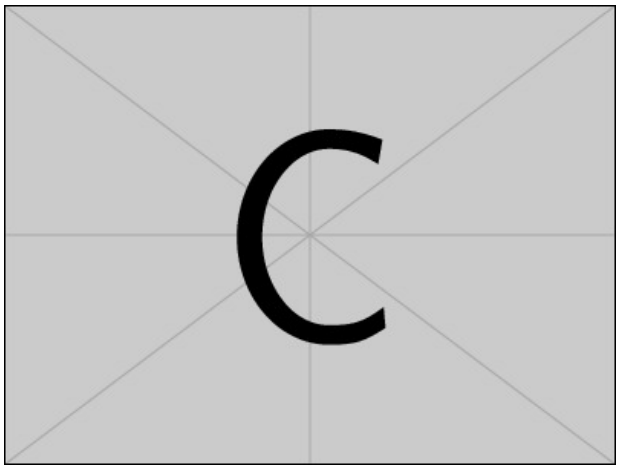
Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.



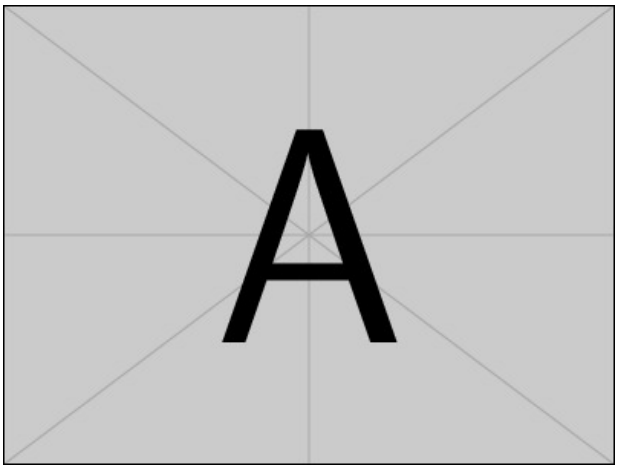
(a) First subfigure



(b) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

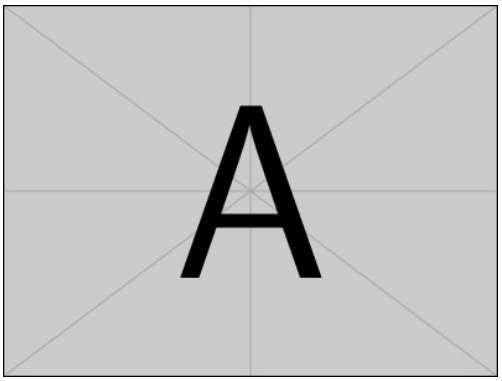


(c) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

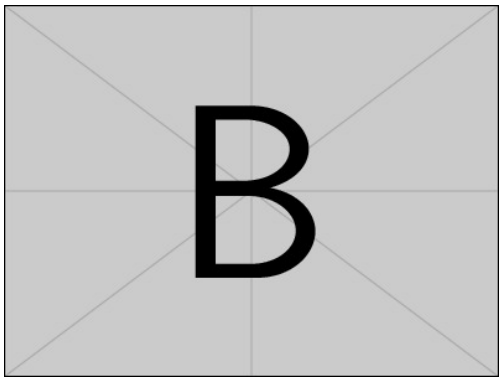


(d) Fourth subfigure, with $\alpha = 0.5$.

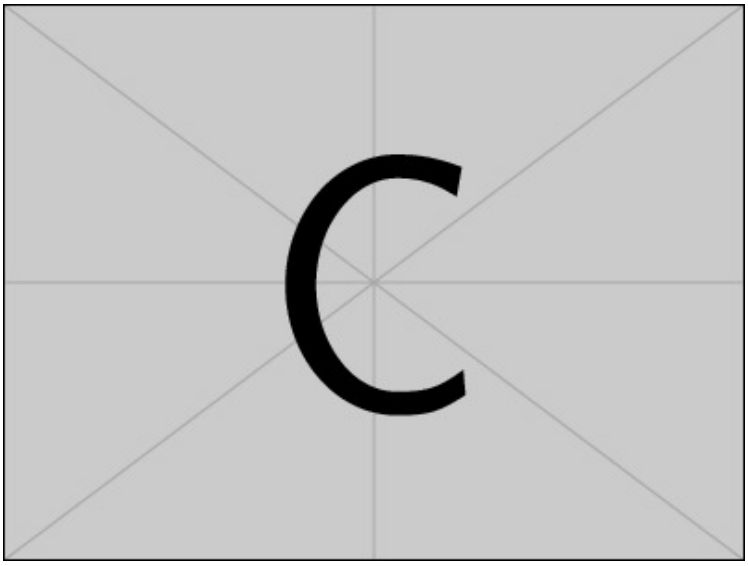
Figure 1.4: A figure with four subfigures: two above two.



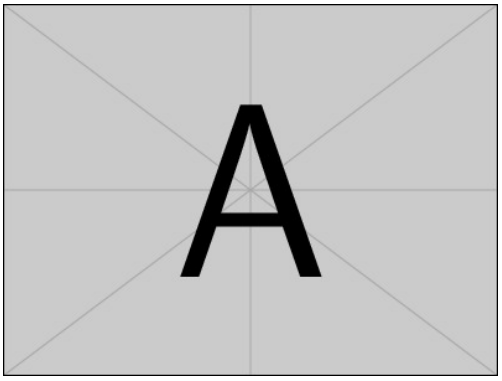
(a) Top left



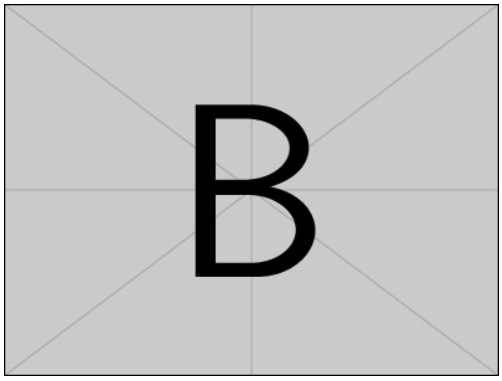
(b) Top right, with a longer caption: postremo aliquos futuros suspicor, qui me ad alias litteras vocent.



(c) The larger middle subfigure, which refers to Figs. 1.5(a) and 1.5(e).



(d) Bottom left, $x^2 + y^2 = r^2$.



(e) Bottom right

Figure 1.5: A figure with five subfigures: two above, one larger in the middle, and two below.

Nomenclature for Chapter 1

Roman letters

- C material curve
- $\mathbf{r}$ material position [m]
- $\mathbf{u}$ velocity [m s^{-1}]

Greek letters

- Γ circulation [m^2 s^{-1}]
- ρ mass density [kg m^{-3}]
- $\boldsymbol{\omega}$ vorticity [s^{-1}]

Appendix A

Multipage and sideways tables

A.1 A multipage table of numbers

This example uses the `longtable` package. The values in the table are the one-term coefficients for one-dimensional, transient heat conduction, $\theta = A_1 f_1 \exp(-\lambda_1^2 \mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2 \mathrm{Fo})$.

Table A.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [21].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770

Table A.1: (continued)

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

A.2 A sideways table

Table A.2 uses the `sidewaystable*` environment from the `rotating` package.

Table A.2: Data sets for local heat transfer coefficients in liquids. Boundary condition (B.C.) and r.m.s. freestream turbulence (u'_r/u_∞) are as indicated.

Authors	**B.C.**	**Range of Re_x**	u'_r/u_∞	**Comments**
Data for Water				
Hollingsworth, 1989	UWT	$\approx 5 \times 10^5$	0.5 – 1.5%	Data for Pr = 5.93 ± 0.13 with 4 K b.l. temperature difference. Boundary layer tripped ahead of plate, with a substantial unheated section; two locations measured in uncurved channel. Plate temperature controlled by rows of electrically heated strips. Data analysis focuses on enthalpy thickness Reynolds number. Data were used to calibrate a numerical model, leading to a correlation: $\mathrm{St} = 0.02426\ \mathrm{Re}_x^{-n}\mathrm{Pr}^{-0.895}$ with $n = 0.1879\ \mathrm{Pr}^{-0.18}$ for $0.7 \leqslant \mathrm{Pr} \leqslant 8$. Measurement uncertainties (2σ): St ± 3.3%; Re_x ± 1.6%.
Žukauskas & Šlančiauskas, 1987 summary	UHF	4.0×10^3 – 4.4×10^6	∼ 1.2%	Data for Prandtl numbers of 2.95, 5.4, and 6.6. Stated uncertainty in heat transfer is up to ±10%, being largest for higher fluxes. Background turbulence measured at inlet to test section. For Pr = 5.4: Run 17, ΔT is 7–8 K for the turbulent and transitional data, with the laminar points rising to 17 K; Run 18, $6.1 \leqslant \Delta T \leqslant 7.5$ K; Table 25, ΔT is 7–8 K for the turbulent and transitional data, again rising to 15–17 K in the laminar region. For Pr = 6.6: Run 7, $1.3 \leqslant \Delta T \leqslant 4.2$ K; Run 8, $6.5 \leqslant \Delta T \leqslant 12.5$ K. For Pr = 2.95: $5.4 \leqslant \Delta T \leqslant 8.6$ K.
Data for higher Pr				
Žukauskas & Šlančiauskas, 1987 summary	UHF	1.2×10^4 – 7.7×10^5	∼ 0.7%	Data for “transformer oil” at several Prandtl numbers (55–257). Uncertainty in heat transfer stated as ±5%. Background turbulence measured at inlet to test section. Many of these data embed significant property variation across the boundary layer. (An additional data set is given for cooling hot glycerin at higher Pr, but with even stronger property variations.)

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