The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1876

© 1876 Silas W. Holman. All rights reserved.

The author hereby grants to MIT a nonexclusive, worldwide, irrevocable, royalty-free license to exercise any and all rights under copyright, including to reproduce, preserve, distribute and publicly display copies of the thesis, or release the thesis under an open-access license.

Authored by: Silas W. Holman
Department of Physics
May 18, 1876

Certified by: Edward C. Pickering
Professor of Physics, Thesis Supervisor

Accepted by: Primus Castor
Professor of Log Dams
Undergraduate Officer, Department of Physics

THESIS COMMITTEE

THESIS SUPERVISOR

Edward C. Pickering
Professor of Physics
Department of Physics

THESIS READERS

Marcus Gavius Apicius
Professor of Cooking Arts
Department of Food Science

Marie-Antoine Carême
Professor of Haute Cuisine
Department of Food Science

Miles Gloriosus
Professor of Personal Pronouns
Department of Rhetoric

The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
on May 18, 1876 in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

[1] Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

Contents

List of Figures 7
List of Tables 8

1 Introduction **9**
1.1 A section discussing the first issue: J/ψ . . . 9
1.1.1 Subsection eqn. (1.2) . . . 10
1.2 Description of our paradigm . . . 11
1.2.1 Conversion to a metaheuristic . . . 11
1.3 Other generalizations . . . 13
1.3.1 The most general case . . . 13
1.4 Baroclinic generation of vorticity . . . 14
1.5 Further examples of multipanel figures . . . 15
1.6 Summary . . . 15
Nomenclature . . . 16

A Multipage and sideways tables **20**
A.1 A multipage table of numbers . . . 20
A.2 A sideways table . . . 21

References 23

List of Figures

1.1 A figure with two subfigures 10
1.2 Caption text [8]. 12
1.3 A figure with three subfigures: two above one. 17
1.4 A figure with four subfigures: two above two. 18
1.5 A figure with five subfigures: two above, one larger in the middle, and two below. . 19

List of Tables

1.1 The error function and complementary error function 15
1.2 Table with more complicated columns: $\Delta T_m = \delta\theta/\Theta_0$ 15
1.3 Brinkman number at which liquid dissipation makes < 5% or < 1% contribution to θ_b or q_w from the Graetz solution at the points where θ_b equals 0.05, 0.02, or 0.01 16

A.1 One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [21]. 20
A.2 Data sets for local heat transfer coefficients in liquids. Boundary condition (B.C.) and r.m.s. freestream turbulence (u'_r/u_∞) are as indicated. 22

Chapter 1

Introduction

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

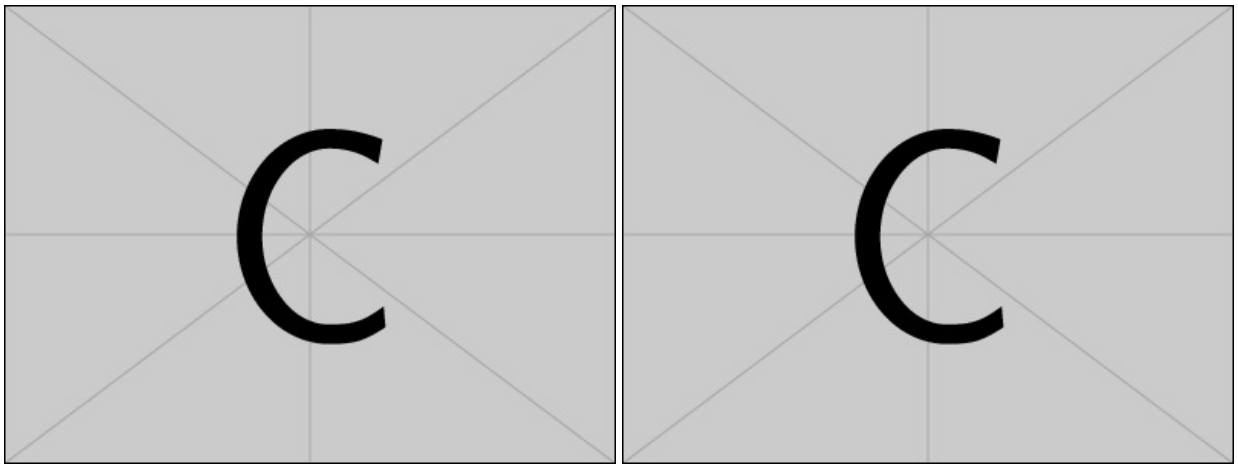
Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris. Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi, etsi sit elegans, personae tamen et dignitatis esse negent [1–5].

1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [6,7].

$$\frac{\partial}{\partial t}\left[\rho\left(e+|\vec{u}|^2/2\right)\right]+\nabla\cdot\left[\rho\left(h+|\vec{u}|^2/2\right)\vec{u}\right]=-\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\left(d_{ji}u_i\right) \tag{1.1}$$

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.



(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Second subfigure

Figure 1.1: A figure with two subfigures

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula. And more citations [7,8]. Then we write some more and include our citations [9–11]. The configuration is shown in Figs. 1.1(a) and 1.1(b).

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1 Subsection eqn. (1.2)

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec

luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

A subsubsection

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1, \varepsilon_1)} & 0 & \cdots & \cdots & \cdots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2, \varepsilon_1)} & \frac{\varphi}{(\varphi_2, \varepsilon_2)} & 0 & \cdots & \cdots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3, \varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3, \varepsilon_2)} & \frac{\varphi}{(\varphi_3, \varepsilon_3)} & 0 & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1}, \varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1}, \varepsilon_2)} & \cdots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1}, \varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1}, \varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n, \varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n, \varepsilon_2)} & \cdots & \cdots & \frac{\varphi k_{n,n-1}}{(\varphi_n, \varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n, \varepsilon_n)} \end{pmatrix} \tag{1.2}$$

1.2 Description of our paradigm

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. No dissertation is complete without footnotes.[1,2,3]

1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id,

[1] First footnote: $a_h = F_m$. See section 1.4.

[2] Another interesting detail.

[3] And another really important idea to have in mind [12–17].

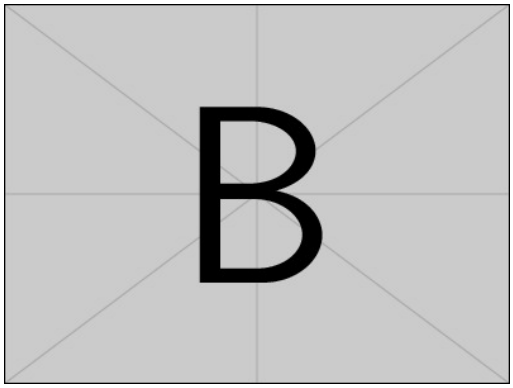


Figure 1.2: Caption text [8].

sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further in section 1.4 and Refs. [18,19].

Subsection

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Paragraph Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Subparagraph Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

1.3 Other generalizations

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will be unambiguous [20].

1.3.1 The most general case

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl. But, according to Lord Brouncker (1655),

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cfrac{9^2}{2 + \cdots}}}}} \tag{1.3}$$

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application of Stokes theorem leads to the *Kelvin–Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\oint_{\mathscr{C}} \mathbf{u}\cdot d\mathbf{r} \tag{1.4}$$

$$= \oint_{\mathscr{C}} \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{r} + \underbrace{\oint_{\mathscr{C}} \mathbf{u}\cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \tag{1.5}$$

$$= \iint_{\mathscr{S}} \nabla\times\frac{D\mathbf{u}}{Dt}\cdot d\mathbf{A} \tag{1.6}$$

$$= \iint_{\mathscr{S}} \nabla p\times\nabla\left(\frac{1}{\rho}\right)\cdot d\mathbf{A} \tag{1.7}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom currents going oppositely.

Weak shock wave A weak shock wave travelling in a perfect gas is described by the following Burgers equation derived from approximations due to G. I. Taylor (1910)

$$u_t + \left(a_0 + \tfrac{1}{2}(\gamma+1)u\right)u_x = \frac{\delta}{2}u_{xx} \tag{1.8}$$

Here, γ is the specific heat capacity ratio, δ is the diffusivity of sound, u_1 is the downstream speed, $u_2 < u_1$ is the upstream speed, and a_0 is the undisturbed sound speed. The travelling-wave, or Taylor shock, solution of this equation is

$$\frac{u-u_2}{u_1-u_2} = \frac{1}{2}\left[1-\tanh\left(\frac{\gamma+1}{4\delta}(u_1-u_2)(x-ct)\right)\right] \tag{1.9}$$

where $c = a_0 + (\gamma+1)(u_1+u_2)/4$. This solution exists for a compressive wave, $u_1 > u_2$.

Boundary layer model K. O. Friedrichs introduced the following model equation to illustrate a boundary layer (1941):

$$\varepsilon\frac{d^2y}{dx^2} + \frac{dy}{dx} = 1+2x, \quad \text{with } y(0)=0 \text{ and } y(1)=1 \tag{1.10}$$

As $\varepsilon \to 0$, the highest derivative is lost: the equation becomes first order and cannot meet two boundary conditions. This fact results in a boundary layer at one side of the domain, with a thickness $\mathcal{O}(\varepsilon)$. Matching the outer and inner expansions gives the uniform approximation

$$y_{\text{unif}} = e^{-x/\varepsilon} - 1 + x + x^2 + 2\varepsilon\left[(1-x) - e^{-x/\varepsilon}\right], \tag{1.11}$$

with $y - y_{\text{unif}} = \mathcal{O}(\varepsilon^3)$ uniformly on $0 \le x \le 1$ as $\varepsilon \to 0$. As it happens, all higher-order terms of both expansions vanish, so y_{unif} is correct to all orders in ε. A comparison to the exact solution shows the error to be subdominant, $\mathcal{O}(e^{-1/\varepsilon})$.

Table 1.1: The error function and complementary error function

x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$	x	$\mathrm{erf}(x)$	$\mathrm{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11979
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Table 1.2: Table with more complicated columns: $\Delta T_m = \delta\theta/\Theta_0$

Experiment	u [m/s]	T [°C]
The first test we ran this morning	124.3	68.3
The second test we ran this morning	82.50	103.46
Our competitor's test	72.321	141.384

1.5 Further examples of multipanel figures

Examples are given in Figs. 1.3, 1.4, and 1.5. Or consider Figs. 1.4(a), 1.4(b), 1.4(c), and 1.4(d).

1.6 Summary

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque

Table 1.3: Brinkman number at which liquid dissipation makes < 5% or < 1% contribution to θ_b or q_w from the Graetz solution at the points where θ_b equals 0.05, 0.02, or 0.01

			|Br|			
θ_b	x^+	$q_w D_h/k\Delta T$	5% of θ_b	1% of θ_b	5% of q_w	1% of q_w
0.05	0.09621	0.3770	3.646×10^{-3}	7.292×10^{-4}	1.571×10^{-3}	3.142×10^{-4}
0.02	0.1266	0.1508	1.458×10^{-3}	2.917×10^{-4}	6.284×10^{-4}	1.257×10^{-4}
0.01	0.1496	0.07541	7.292×10^{-4}	1.458×10^{-4}	3.142×10^{-4}	6.284×10^{-5}

habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

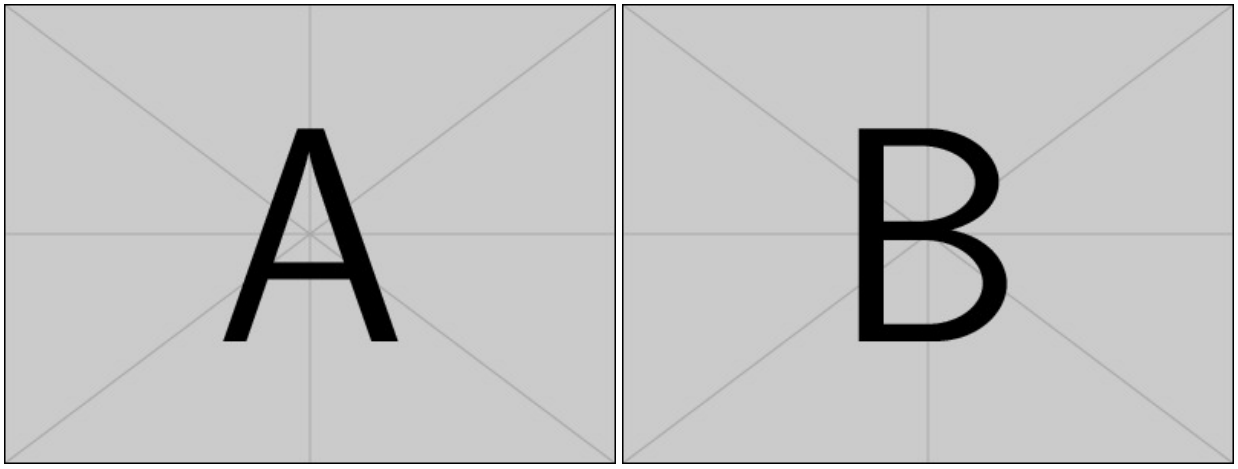
Nomenclature for Chapter 1

Roman letters

- $\mathscr{C}$ material curve
- $\mathbf{r}$ material position [m]
- $\mathbf{u}$ velocity [m s^{-1}]

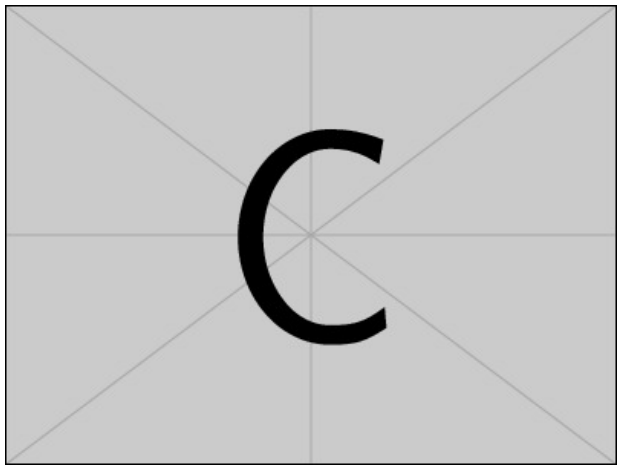
Greek letters

- Γ circulation [m^2 s^{-1}]
- ρ mass density [kg m^{-3}]
- $\boldsymbol{\omega}$ vorticity [s^{-1}]



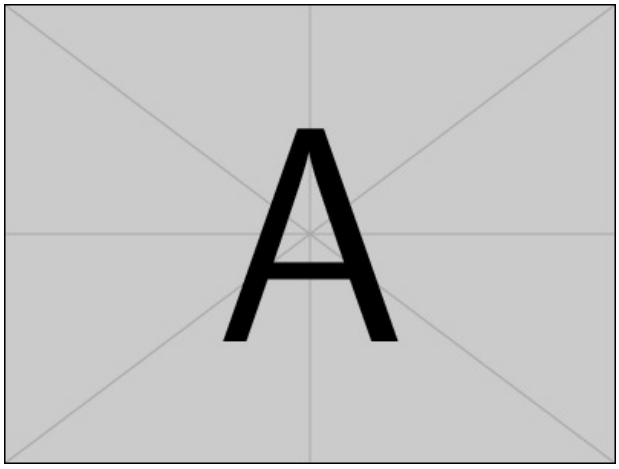
(a) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

(b) Short caption

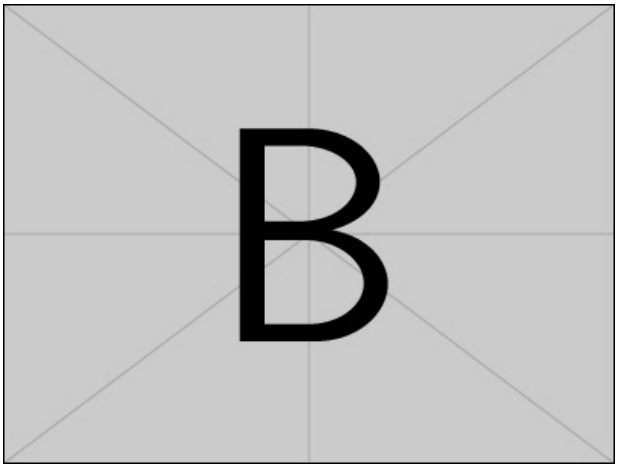


(c) Caption with math, $\mathrm{Re}_d = 100$, and a reference to Fig. 1.3(a).

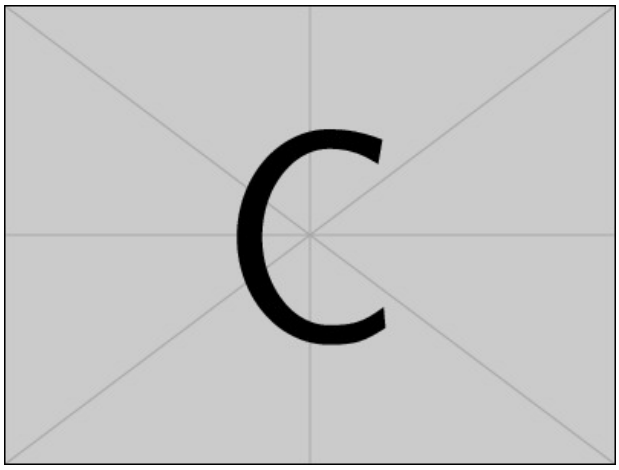
Figure 1.3: A figure with three subfigures: two above one.



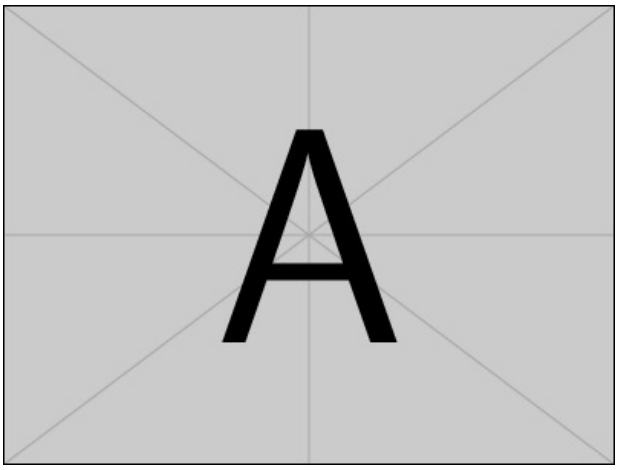
(a) First subfigure



(b) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

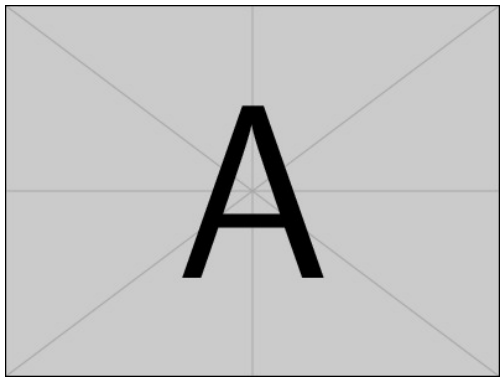


(c) Postremo aliquos futuros suspicor, qui me ad alias litteras vocent, genus hoc scribendi.

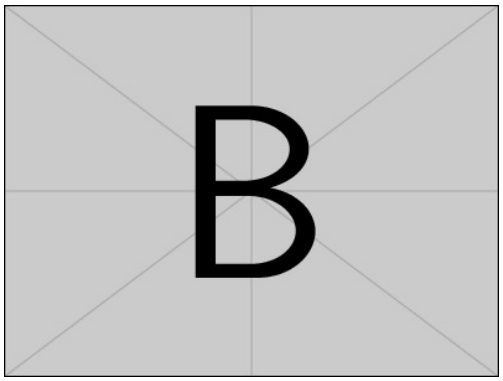


(d) Fourth subfigure, with $\alpha = 0.5$.

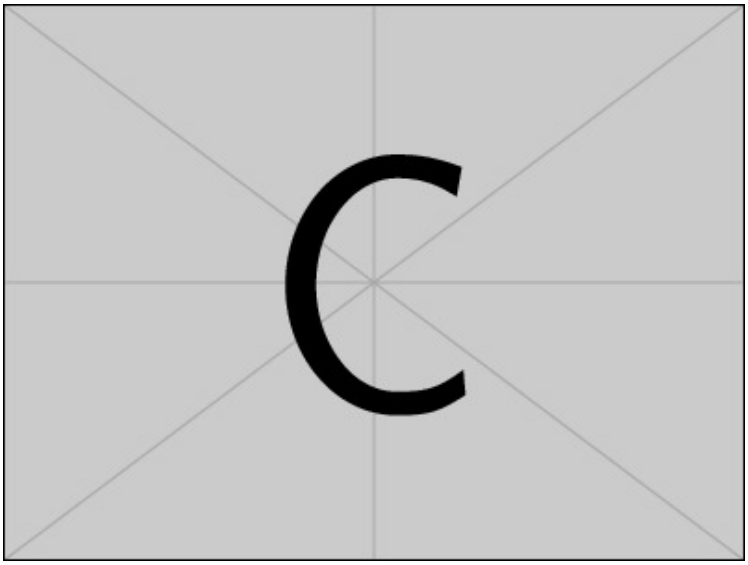
Figure 1.4: A figure with four subfigures: two above two.



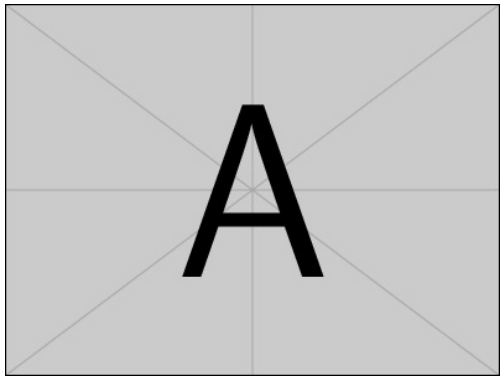
(a) Top left



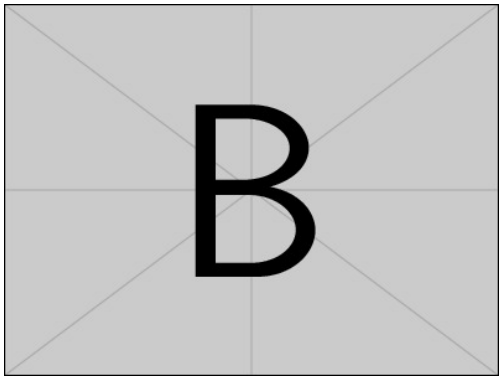
(b) Top right, with a longer caption: postremo aliquos futuros suspicor, qui me ad alias litteras vocent.



(c) The larger middle subfigure, which refers to Figs. 1.5(a) and 1.5(e).



(d) Bottom left, $x^2 + y^2 = r^2$.



(e) Bottom right

Figure 1.5: A figure with five subfigures: two above, one larger in the middle, and two below.

Appendix A

Multipage and sideways tables

A.1 A multipage table of numbers

This example uses the `longtable` package. The values in the table are the one-term coefficients for one-dimensional, transient heat conduction, $\theta = A_1 f_1 \exp(-\lambda_1^2 \mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2 \mathrm{Fo})$.

Table A.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [21].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739

Table A.1: (continued)

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

A.2 A sideways table

Table A.2 uses the `sidewaystable*` environment from the `rotating` package.

Table A.2: Data sets for local heat transfer coefficients in liquids. Boundary condition (B.C.) and r.m.s. freestream turbulence (u'_r/u_∞) are as indicated.

Authors	**B.C.**	**Range of Re_x**	u'_r/u_∞	**Comments**
Data for Water				
Hollingsworth, 1989	UWT	$\approx 5 \times 10^5$	0.5 – 1.5%	Data for Pr = 5.93 $\pm$ 0.13 with 4 K b.l. temperature difference. Boundary layer tripped ahead of plate, with a substantial unheated section; two locations measured in uncurved channel. Plate temperature controlled by rows of electrically heated strips. Data analysis focuses on enthalpy thickness Reynolds number. Data were used to calibrate a numerical model, leading to a correlation: $\mathrm{St} = 0.02426\ \mathrm{Re}_x^{-n}\mathrm{Pr}^{-0.895}$ with $n = 0.1879\ \mathrm{Pr}^{-0.18}$ for $0.7 \leqslant \mathrm{Pr} \leqslant 8$. Measurement uncertainties (2σ): St $\pm$ 3.3%; Re_x $\pm$ 1.6%.
Žukauskas & Šlančiauskas, 1987 summary	UHF	$4.0 \times 10^3 - 4.4 \times 10^6$	~ 1.2%	Data for Prandtl numbers of 2.95, 5.4, and 6.6. Stated uncertainty in heat transfer is up to $\pm$10%, being largest for higher fluxes. Background turbulence measured at inlet to test section. For Pr = 5.4: Run 17, ΔT is 7–8 K for the turbulent and transitional data, with the laminar points rising to 17 K; Run 18, $6.1 \leqslant \Delta T \leqslant 7.5$ K; Table 25, ΔT is 7–8 K for the turbulent and transitional data, again rising to 15–17 K in the laminar region. For Pr = 6.6: Run 7, $1.3 \leqslant \Delta T \leqslant 4.2$ K; Run 8, $6.5 \leqslant \Delta T \leqslant 12.5$ K. For Pr = 2.95: $5.4 \leqslant \Delta T \leqslant 8.6$ K.
Data for higher Pr				
Žukauskas & Šlančiauskas, 1987 summary	UHF	$1.2 \times 10^4 - 7.7 \times 10^5$	~ 0.7%	Data for "transformer oil" at several Prandtl numbers (55–257). Uncertainty in heat transfer stated as $\pm$5%. Background turbulence measured at inlet to test section. Many of these data embed significant property variation across the boundary layer. (An additional data set is given for cooling hot glycerin at higher Pr, but with even stronger property variations.)

References

[1] D. K. Edwards. "Radiative Transfer Characteristics of Materials." *ASME J. Heat Transfer*, **91**(1), Feb. 1969, pp. 1–15. DOI: 10.1115/1.3580108.

[2] E. T. Whittaker and G. N. Watson. *A Course of Modern Analysis*. 3rd ed. Cambridge, UK: Cambridge University Press, 1920. URL: https://archive.org/details/courseofmodernan00whit.

[3] J. T. Kirk. *Decline and Fall of the Romulan Empire*. 7th ed. Humankind's Greatest Writings 23. T'Paal: Vulcan Free Press, 2288. To appear.

[4] W. S. Churchill. *The Gathering Storm*. In: *The Second World War*. Vol. 1. Boston: Houghton Mifflin Co., 1948. Chap. 5, "The Locust Years", pp. 66–89. URL: https://search.worldcat.org/title/3025315.

[5] J. W. Gibbs. "On the Form of the Teeth of Wheels in Spur Gearing." PhD thesis. New Haven, CT: Yale University, 1863.

[6] C. Fong. *Analytical Methods for Squaring the Disc*. Sept. 2015. arXiv: 1509.06344.

[7] M. Sharpe. *New TX Font Package*. Version 1.71. Comprehensive TEX Archive Network, Mar. 2022. URL: https://ctan.org/pkg/newtx (visited on 05/29/2023).

[8] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, P. Alken, M. Booth, F. Rossi, and R. Ulerich. *GNU Scientific Library*. Version 2.4. Free Software Foundation. Boston, MA, 2017. URL: https://www.gnu.org/software/gsl/ (visited on 12/12/2018).

[9] J. Swaminathan, R. L. Stover, E. W. Tow, D. M. Warsinger, and J. H. Lienhard. "Effect of Practical Losses on Optimal Design of Batch RO Systems." In: *Proceedings of IDA World Congress on Desalination and Water Reuse* (São Paulo, Brazil, Oct. 15–20, 2017). International Desalination Association, Oct. 2017, IDA17WC-58334. HDL: 1721.1/111971.

[10] F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, B. V. Saunders, H. S. Cohl, and M. A. McClain, eds. *NIST Digital Library of Mathematical Functions*. Version 1.2.8. National Institute of Standards and Technology, Sept. 15, 2026. URL: https://dlmf.nist.gov/ (visited on 11/12/2026).

[11] American Mathematical Society and The LATEX Project. *User's Guide for the amsmath Package*. Version 2.1. Comprehensive TEX Archive Network. Feb. 2020. URL: https://ctan.org/pkg/amsmath (visited on 05/29/2023).

[12] W. C. Reynolds, W. M. Kays, and S. J. Kline. *Heat Transfer in the Turbulent Incompressible Boundary Layer. I—Constant Wall Temperature*. NASA Memorandum No. 12-1-58W. Washington, DC: National Aeronautics and Space Administration, Dec. 1958. HDL: 2060/19980228020.

[13] F. H. Clauser. “The Turbulent Boundary Layer.” In: *Advances in Applied Mechanics*. Ed. by H. L. Dryden and T. von Kármán. Vol. 4. New York: Academic Press, 1956, pp. 1–51. DOI: 10.1016/S0065-2156(08)70370-3.

[14] J. H. Lienhard. “Heat Transfer in Flat-Plate Boundary Layers: A Correlation for Laminar, Transitional, and Turbulent Flow.” *ASME J. Heat Transfer*, **142**(6), 061805, June 2020. DOI: 10.1115/1.4046795.

[15] H. Johnson, ed. *Title of Edited Book*. New York, NY: John Wiley & Sons, Inc., 1980.

[16] H. W. Johnson, ed. *The Title of the Proceedings* (City, State, Apr. 27–28, 1965). Vol. 2. Organization. 1965. URL: https://news.mit.edu/2009/obit-johnson.

[17] S. van der Walt and N. Smith. *mpl Colormaps*. Berkeley Institute for Data Science, Sept. 2015. URL: https://bids.github.io/colormap/ (visited on 11/12/2026).

[18] L. Euler. “De summis serierum reciprocarum.” *Commentarii Academiae Scientiarum Petropolitanae*, **7**, 1740, pp. 123–134. First communicated to Daniel Bernoulli in 1734 and read before the St. Petersburg Academy in December 1735. Published in English as *On the sums of series of reciprocals*. Trans. from the Latin by J. Bell. June 2005. arXiv: math/0506415 [math.HO].

[19] J. B. J. Fourier. *Théorie analytique de la chaleur*. Paris: Firmin Didot, Père et Fils, 1822. URL: https://archive.org/details/thorieanalytiq00four. Published in English as *The Analytical Theory of Heat*. Trans. from the French and annot. by A. Freeman. Cambridge, UK: Cambridge University Press, 1878. URL: https://archive.org/details/analyticaltheory00fourrich.

[20] J. I. Montijano, M. Pérez, L. Rández, and J. L. Varona. “Numerical Methods with LuaLaTeX.” *TUGboat*, **35**(1), Jan. 2014, pp. 51–56. URL: https://tug.org/TUGboat/tb35-1/tb109montijano.pdf.

[21] H. D. Baehr and K. Stephan. *Heat and Mass Transfer*. Trans. from the German by N. J. Park. Berlin: Springer-Verlag, 1998. ISBN: 3-540-63695-1.